Cavity Quantum Electrodynamics for Continuous Weak Measurement

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Continuous weak measurement of cavity quantum electrodynamics systems enables one to gain information about the system under measurement without destroying the system state prior to measurement. In this review, I will give a brief introduction to cavity quantum electrodynamics and measurement theory and review works that apply cavity-based weak measurement as a means to study the fundamental physics of measurement-induced decoherence and to extract information about the system for quantum feedback and control.

I. INTRODUCTION

Cavities have gradually developed to become a key part of an experimentalist's toolkit in fields such as quantum optics, quantum simulation, and quantum information [1]. Following the development of the Jaynes-Cummings model [2], which describes the dynamics of a simple two-level atom coupled to a single mode of a cavity, and the pioneering experimental observation of quantum collapse and revival of a single atom coupled to an optical cavity [3], a wide variety of works in both experiment and theory have applied the cavity as an integral device for the study of both fundamental and applied physics [4–6]. Notably, Serge Haroche was awarded the Nobel prize in 2012 for the development of cavity quantum electrodynamics (CQED) and the realization of Schrodinger cat states in a CQED system [7].

In this review, I will focus on the cavity as a tool for continuous weak measurement[8]. The role of the cavity as a means for weak measurement can be applied in a variety of ways, from exploring the dynamics of nonadiabatic processes to the investigation of non-equilibrium quantum systems [9]. I will begin by discussing some basic theory of cavities and cavity quantum electrodynamics, then go on to define and discuss weak measurements, and then review works that investigate the theory of cavity weak measurement and apply cavity weak measurement for quantum feedback and control.

II. CAVITY QUANTUM ELECTRODYNAMICS

A cavity can be thought of as any arrangement of mirrors that act as a resonator for various modes of light. Such devices are used in a wide variety of areas, such as amplification, interferometery, and lasing. The modes of the resonator are dictated by the focal length of the mirrors. One of the most basic descriptions of a cavity is the Fabry-Perot cavity, which simply consists of two mirrors in the configuration shown in figure 1.

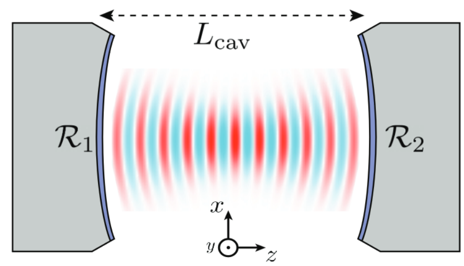

FIG. 1. Schematic of a simple Fabry-Perot cavity. Figure taken from Gerber, Cavity Quantum Electrodynamics with a Locally Addressable Gas [10].

The two flat outer edges of the cavity are coated to be highly anti-reflective, while the two inner sides are coated to be highly reflective (R_1 and R_2 in figure 1). A drive light is sent into the cavity and repeatedly reflects and transmits in this two mirror systems. Let us reduce this system down to two flat 1-D mirrors and examine how we might model such a scenario. For two identical mirrors, let us define reflection and transmission coefficients for each mirror r and t, then define a total reflection and transmission to be $R = r^2$ and $T = t^2$, where $R + T = 1$ (Note that two identical mirrors are used here for the sake of simplicity. For usual experimental applications, mirrors often have differing transmission and reflection coefficients). For an incident electromagnetic wave E_i, we can then compute the transmission and reflection as follows:

$$E_T = TE_ie^{-ikL} \tag{1}$$
$$+ T(E_ie^{-2ikL}R)E_ie^{-ikL} \tag{2}$$
$$+ TE_ie^{-ikL}(E_ie^{-2ikL}R)^2 \tag{3}$$
$$+ \cdots \tag{4}$$
$$= \frac{TE_ie^{-ikL}}{1 - RE_ie^{-2ikL}} \tag{5}$$

where L is the length of the cavity and we note that the field accumulates a phase factor for every additional round trip [11]. A similar procedure can be applied to the reflection to find that the reflected light is of form

$$E_R = \frac{\sqrt{R}E_i(1 - e^{-2ikL})}{1 - Re^{-2ikL}}. \tag{6}$$

We can observe the ratio of intensities between input and output light by simply looking at the square of the electric field. We find that

$$\frac{I}{I_0} = \left|\frac{Te^{-ikL}}{1 - Re^{-2ikL}}\right|^2 = \frac{T^2}{1 + R^2 - 2R\cos kL} \quad (7)$$

and from this, it naturally follows that the transmission will be maximized for $\cos kL = 1$, which dictates the modes of the cavity. Applying the relation between wave number and wavelength, we find that the modes of the cavity are given by

$$\lambda = \frac{2L}{n} \quad (8)$$

where n is some positive integer. This can also be expressed in terms of the frequency

$$\omega = \frac{nc}{2L}. \quad (9)$$

We thus observe transmission peaks indexed in the above manner, as shown in figure 2. The frequency / wavelength separation between two peaks is called the free spectral range (FSR) and is defined to be

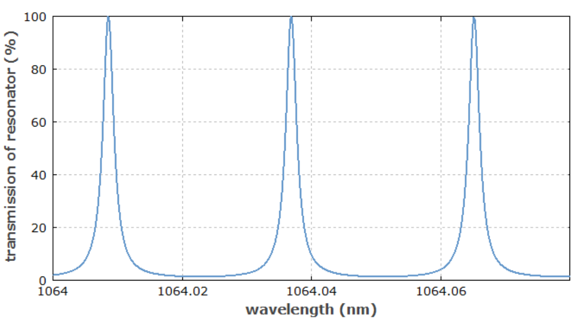

FIG. 2. Sample data for the free spectral range of a cavity. Image taken from RP Photonics.

$$FSR = \frac{c}{2L} \quad (10)$$

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Another useful parameter to note here is the cavity finesse $\mathcal{F}$, which can be described to be the ratio of the cavity's FSR to its energy decay rate. For a non-idealized Fabry-Perot cavity, the mirrors will not be perfectly transmissive and reflective, and there will be some loss L such that $R + T + L = 1$. If we take this loss into account, we can define the finesse to be

$$\mathcal{F} = \frac{FSR}{\kappa} = \frac{\pi}{T + 2L} \quad (11)$$

where κ is the cavity decay parameter and represents the light that leaks out of the cavity. For some time τ, κ can be defined to be

$$\kappa = 2(T + L)\tau. \quad (12)$$

A. Cavity Quantum Electrodynamics

Cavity quantum electrodynamics describes the study of light-matter interactions within a cavity. The simplest toy model for such systems is the Jaynes-Cumming Hamiltonian [12], which describes the coupling between a two-level system with ground state $|g\rangle$ and excited state $|e\rangle$ with a harmonic oscillator in the rotating frame. Dissipative effects such as spontaneous emission or cavity input/output are entirely ignored. The Jaynes-Cumming Hamiltonian is given to be

$$H = \hbar\omega_o a^\dagger a + \hbar\omega_a \sigma^\dagger \sigma + \hbar g(a^\dagger \sigma + a\sigma^\dagger) \quad (13)$$

where ω_0 is the oscillator resonance frequency, a and $a^\dagger$ are the harmonic oscillator creation and annihilation operators, ω_a is the atomic energy splitting, σ and $\sigma^\dagger$ are the atomic raising and lowering operators, and g is the cavity coupling parameter. The Jaynes-Cumming model can be written to be $H = H_A + H_F + H_{int}$ where H_A describes the Hamiltonian of the two-level atom, H_F denotes the Hamiltonian for the cavity field, and H_{int} describes the interaction between the two. The interaction term can be derived by considering the interaction energy of the atomic dipole with the electric field, given by

$$H_{int} = -E \cdot d \quad (14)$$

where $d = -q\hat{r}$ is the dipole moment of the atom and E denotes the electric field. Parity considerations and rewriting the incident field in terms of the creation and annihilation operators enable us to denote the dipole interaction to be

$$H_{int} = -E \cdot d = g(r)a^\dagger \sigma + g^*(r)a\sigma^\dagger. \quad (15)$$

The Jaynes-Cumming model is schematically shown in figure 3.

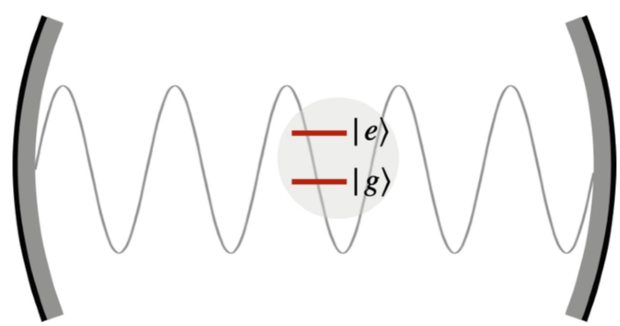

FIG. 3. Schematic of the Jaynes-Cumming model. Figure taken from Meystre, Quantum Optics [13]

For such a system, we also expect to observe a generalized Rabi frequency (for an off-resonant drive field) of form

$$\Omega_n = \sqrt{(n+1)g^2 + \delta^2} \quad (16)$$

where δ describes the detuning of the field from resonance. The atom-cavity systems oscillates back and forth between the states $|g\rangle|n+1\rangle$ and $|e\rangle|n\rangle$ at frequency Ω_n. Note that for the $n=0$ vacuum state, the separation between the eigenvalues is g, which is the vacuum Rabi splitting.

Another parameter of note is the cavity cooperativity, given by

$$C = \frac{4g^2}{\kappa\gamma} \tag{17}$$

where g is the coupling parameter, κ is the cavity decay rate, and γ is the atomic decay rate. Contributions to κ typically arise from leakage of the mode out of the cavity, while γ consists of spontaneous emission from the atom into free space. The cooperativity can be thought of as the ratio between coherent and decoherent effects, and for atom-cavity systems where $g \gg \kappa, \gamma$, the decoherence effects, the system is said to be in the strong coupling regime.

III. MEASUREMENT

Possible measurements of a quantum system can be described by the set of measurement operators $\{A_n\}, n = 1 \dots N$, that satisfy the relation $\sum_n A_n^\dagger A_n = I$, where n labels possible measurement outcomes [14]. For some system S described with the density matrix $\rho^{(S)}$, the state of the system after obtaining the measurement result n can be described by $\tilde{\rho_n}^{(S)}$, described by

$$\tilde{\rho}_n^{(S)} = \frac{A_n \rho_n^{(S)} A_n^\dagger}{p_n} \tag{18}$$

where p_n, the probability of obtaining result n, is described to be

$$p_n = Tr[A_n^\dagger A_n \rho^{(S)}]. \tag{19}$$

This formalism follows very closely with classical Bayesian inference, where the state of the system following measurement is given by the posterior distribution $P(n|j)$, described by

$$P(n|j) = \frac{P(j|n)P(n)}{\mathcal{N}} \tag{20}$$

where $P(j|n)$ represents the likelihood function for the system, $P(n)$ is the prior probability distribution, and $\mathcal{N}$ is the usual Bayesian normalization factor [15]. For a density matrix and measurement operator of forms

$$\rho = \sum_n P(n)\,|n\rangle\langle n| \tag{21}$$

$$A_j = \sum_n A(j,n)\,|n\rangle\langle n| \tag{22}$$

equation 18 can be rewritten to be

$$\tilde{\rho_j} = \frac{A_j \rho A_j^\dagger}{Tr[A_j^\dagger A_j \rho]} = \frac{\sum_n A^2(j,n)P(n)\,|n\rangle\langle n|}{\mathcal{N}} \tag{23}$$

where $A^2(j,n)$ takes on the role of the likelihood in classical Bayesian inference.

A. Projective Measurement

Let us first consider a "strong" (projective) measurement. For an initially mixed state, a projective measurements projects onto a single pure states, thus disturbing the original quantum system and destroying coherence. We can examine this phenomena in greater detail through the density matrix formalism. For some basis $|m\rangle$, there exist projection operators defined to be

$$P_m = |m\rangle\langle m| \tag{24}$$

that take our initial state and project it onto a single basis state. In the density matrix formalism, application of a projective measurement on some system S described by the density matrix

$$\rho^{(S)} = \sum_i p_i \left|n^{(s)}\right\rangle\left\langle n^{(s)}\right|, \tag{25}$$

where p_i denotes the probability of finding the system in the associated state vector, yields a post-projection density matrix $\tilde{\rho}$ of form

$$\tilde{\rho}^{(s)} = \frac{P_m \rho^{(s)} P_m}{Tr(P_m \rho^{(s)} P_m)} = |m\rangle\langle m| \tag{26}$$

and we see that we have collapsed the mixture of states in the initial density matrix $\rho^{(s)}$ into a single basis and we have lost information pertaining to the system's premeasurement state.

B. Weak Measurement

Whereas projective measurement projects onto a pure state, weak measurements only provide partial information and retains the mixture of states and can be applied to gain some quantum information from the system under measurement while still retaining its quantum coherence. To experimentally perform a weak measurement, one can weakly couple the system state to the detector and then project it. Such methods have been applied to perform measurements of the continuous quantum dynamics in processes such as spontaneous emission and quantum jumps, [16] as well as perform quantum feedback and continuous error correction [17, 18].

This process is well illustrated with an example of two state system with density matrix

$$\rho = p_0\,|0\rangle\langle 0| + p_1\,|1\rangle\langle 1|\,. \tag{27}$$

We can also describe the measurement operators

$$A_0 = \sqrt{k}\,|0\rangle\langle 0| + \sqrt{1-k}\,|1\rangle\langle 1| \tag{28}$$

$$A_1 = \sqrt{k}\,|1\rangle\langle 1| + \sqrt{1-k}\,|0\rangle\langle 0|\,. \tag{29}$$

We see that for $k=1$ or 0, the operators A_0 and A_1 are just projectors and for $k=0.5$, the operators are simply the identity operator and no knowledge is obtained following measurement. k can thus be thought of as a variable that denotes the "strength" of measurement. For $k \in (\frac{1}{2}, 1)$, the measurement changes the values of p_0 and p_1 but does not collapse the density matrix to a pure state.

An additional framework that is useful to know is the continuous measurement formulation, in which a series of discrete weak measurement are taken at small timesteps dt. For a system with Hamiltonian H where some observable X is being continuously measured, the evolution of the density matrix ρ is given to be a stochastic equation of form

$$d\rho = -i[H,\rho]dt - k[X,[X,\rho]]dt \tag{30}$$

$$+\sqrt{2k}(X\rho + \rho X - 2\langle X\rangle\rho)dW \tag{31}$$

where k is the measurement strength and scales with the rate at which measurement extracts information and dW is the Wiener noise. The sequence of states that arises over the course of the evolution of the density matrix is commonly referred to as the quantum trajectory. Such a formalism is especially useful in the cavity QED context as experiments often involve continuous measurements of the cavity field.

C. Cavity Weak Measurement: Photodetector System

For a better understanding of weak measurement with a cavity, I will now discuss the theoretical framework for detection of photons leaking out of a cavity. For a cavity decay rate of κ, the photon leakage rate is given to be $\kappa\langle a^\dagger a\rangle$, which is just the decay rate multiplied by the average photon number. The instantaneous photon count rate $\lambda(t)$ is then given to be

$$\lambda(t) = \kappa Tr[a^\dagger a\rho(t)] \tag{32}$$

and the measurement operators are

$$A_0 = a\sqrt{\kappa dt} \tag{33}$$

$$A_1 = 1 - \frac{\kappa}{2}a^\dagger a dt. \tag{34}$$

Such a system can then be described by the following master equation

$$d\rho = -\frac{i}{\hbar}[H,\rho]dt - \frac{\kappa}{2}(a^\dagger a\rho + \rho a^\dagger a - 2a^\dagger\rho a) \tag{35}$$

where the Hamiltonian is the typical Hamiltonian for that of light in a cavity, $H = \hbar\omega(a^\dagger a + \frac{1}{2})$ where ω is the angular frequency of the light.

D. Cavity Weak Measurement: Homodyne System

Homodyne detection involves interfering the signal with a local oscillator that is a coherent beam of the same frequency on a beamsplitter and is a common detection technique for experimental cavity systems. A schematic is shown in figure 4.

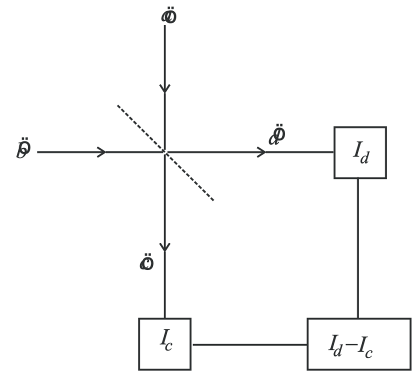

FIG. 4. Schematic of homodyne detection. Input fields a and b enter a beamsplitter and are interfered. The photocurrent of the subsequent outputs c and d can then be measured on a balanced photodetector. Image taken from Gerry and Knight, Introductory Quantum Optics [12].

The detection rate then becomes

$$\kappa\langle\tilde{a}^\dagger\tilde{a}\rangle = \kappa\langle a^\dagger a\rangle - i\kappa\beta\langle a - a^\dagger\rangle + \gamma|\beta|^2 \tag{36}$$

where the annihilation operators a and b describe the input mode of the beamsplitter and $\tilde{a}$ and $\tilde{b}$ describe the output modes of the beamsplitter. Typically, b is chosen to be a coherent state with amplitude α, and $\beta = \alpha\sqrt{1-\eta}$ where η describes the transmission of the beamsplitter. The observable that is continuously measured then becomes the phase quadrature operator Y or the amplitude quadrature operator X where

$$X = \frac{a + a^\dagger}{2} \tag{37}$$

$$Y = \frac{-i(a - a^\dagger)}{2}. \tag{38}$$

The stochastic Schrodinger equation describing the state of a cavity under homodyne detection is

$$d\rho = -\frac{i}{\hbar}[H,\rho]dt \tag{39}$$

$$-k(A^\dagger A\rho + \rho A^\dagger A - 2A^\dagger\rho A)dt \tag{40}$$

$$+\sqrt{2k}(A\rho + \rho A^\dagger - \langle A + A^\dagger\rangle\rho)dW \tag{41}$$

where $A = ia$ and $k = \kappa/2$.

IV. INVESTIGATING CAVITY WEAK MEASUREMENT

A variety of work has been done on use of cavities for weak measurement. As a point of clarification, weak measurement and quantum non-demolition measurement are often used interchangeably in the literature. However, this tends to be an abuse of terminology, as weak measurement typically refers to non-projective measurement, while quantum non-demolition measurement refers to performing a measurement that commutes with the system Hamiltonian. I will focus solely on works that utilize weak measurement.

In the context of cavities, the most commonly treated case is that of an atom or some finite-level system weakly coupled to a dissipative cavity, so that information can be recovered from cavity leakage. Yu et al. demonstrated numerically that in the case of a two-level atom coupled to a cavity, the strength of the measurement, determined by p_0 and p_1, scale with the quantum coherence of the atom [19]. As shown in figure 5, the stronger the measurement, the more coherence is destroyed.

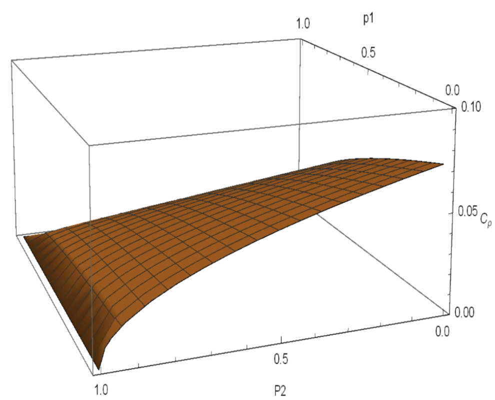

FIG. 5. Scaling of p_0 and p_1 with coherence C_ρ. Figure taken from Yu et al.

Wiseman also gives a thorough theoretical treatment of weak measurements in a cavity, though he expands the theory to weak values (weak values, as described by Aharanov, Albert, and Vaidman [20], are ensemble averaged results of weak measurements) and presents a theoretical model for an experimental system where pre-selection of state preparation and post-selection of measurement value is performed [21]. Wiseman models the experimental set-up of Foster et al. [22] which consists of homodyne detection of cavity leakage from atoms in a weakly driven and damped cavity in the strong coupling regime. In such a system, continuous measurements are made, and thus are described as a series of discrete weak measurements taken with small timesteps δt like the formalism described in section III B. For some random variable X_n and measurement operators M_{xx}, the post-measurement density matrix $\tilde{\rho}$, Wiseman demonstrates that the weak value result can be described in a similar manner to the description given in section III, where

$$\tilde{\rho}_x(t+\delta t) = \frac{M_x \rho(t) M_x^\dagger}{Tr[M_x \rho(t)] M_x^\dagger}. \tag{42}$$

With the introduction of post-selection, where trials are only kept if events of successful state preparation and successful final measurement E and ρ are satisfied, the weak value X_w at some time t can then be found to be

$$_E\langle X_w \rangle_\rho = \frac{\langle E(T)c(t) + c^\dagger(t)E(T)\rangle}{\langle E(T)\rangle} \tag{43}$$

where $E(T)$ is a positive operator that describes the final measurement at time $T = t + \tau$, c is proportional to the lowering operator of the radiative system, and the brackets denote ensemble averaging. This form for the weak value looks very similar to a correlation function that can be derived from homodyne measurements. Further manipulation of this equation yields a simplified form of

$$_E\langle X_w \rangle_\rho = 2\text{Re}\frac{Tr[Ec\rho]}{Tr[E\rho]} \tag{44}$$

For the experiment undertaken by Foster et al., Wiseman derives the measurement operators for finding no photons or one photon in the system E_0, E_1, where

$$E_0 = 1 - \delta t \kappa \eta_c a^\dagger a \tag{45}$$
$$E_1 = \delta t \kappa \eta_c a^\dagger a \tag{46}$$

where κ is the cavity decay rate and η_c is the efficiency of the homodyne detector. The measured weak value $X_w(t)$ is the homodyne current, and for preselection of a system in its stationary state and postselection of a photon being in the system the correlation function of the system can be written to be

$$_{E_1}\langle X_w \rangle_{\rho_{ss}} = 2\text{Re}\frac{Tr[E_1 a \rho_{ss}]}{Tr[E_1(T)\rho_{ss}]} \tag{47}$$

which is analogous to that of equation 44. Wiseman demonstrates that homodyne detection of the cavity field is well described with the theory of continuous weak-measurement.

Within this framework, a number of works have been done on weak measurement through observation of photons leaking out of a dissipative cavity. Laflamme et al. performed a thorough theoretical analysis of the backaction dephasing rate and the measurement rate for a qubit under weak measurement in a nonlinear cavity [23]. They explicitly operate in a weak measurement regime they deem "weak-but-not-too-weak", such that the Hamiltonian can no longer be accurately treated through perturbation theory. They utilize a microwave cavity as their model system, where the qubit is dispersively coupled to the cavity, enabling the qubit state to be monitored through the phase and frequency of emitted microwaves from the cavity. Weak measurements of the qubit state are performed by monitoring the cavity leakage field on a homodyne detector. Laflamme et al. theoretically demonstrated that the efficiency of weak measurements χ can be improved by increasing the atom-cavity coupling parameter beyond the extreme weak-coupling limit, where χ is defined to be $\chi = \Gamma_{meas}/\Gamma_\phi$, where Γ_{meas} is the measurement rate and Γ_ϕ is the backaction dephasing rate. This effect is demonstrated in figure 6.

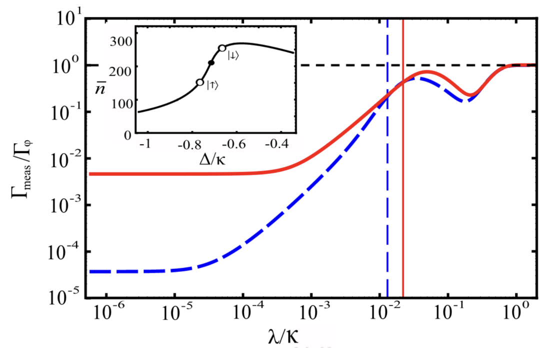

FIG. 6. Efficiency of weak measurement vs. coupling parameter. Figure taken from Laflamme et al. The dashed blue line is the calculation performed for a photon number gain G of 10^3 while the solid red curve is for $G = 10^2$.

V. QUANTUM FEEDBACK

Another realm of possibility that arises from continuous weak measurement with a cavity is quantum feedback, in which the quantum system system of interest is continuously monitored and the information gained from measurement is then used to alter the system Hamiltonian. This procedure can be mathematically described with the density matrix formalism, where the system density matrix $\rho(t)$ can be written to be

$$\rho(t+\Delta t) = \frac{U_j A_j \rho(t) A_j^\dagger U_j^\dagger}{Tr[A_j^\dagger A_j]} \tag{48}$$

where A_j are the usual measurement operators, U_j are a set of unitaries that are applied in response to the measurement result, and j labels the measurement result. Additionally, when feedback is applied in response to some continuous measurement of an operator x, the stochastic master requation is then modified to be

$$d\rho = -\frac{i}{\hbar}\left[H_0 + \sum_n \mu_n(\rho) H_{n,\rho}\right] dt \tag{49}$$

$$- k[x,[x,\rho]]dt + \sqrt{2\eta k}(x\rho + \rho x - 2\langle x\rangle\rho)dW \tag{50}$$

where mu_n index the control parameters, k is the measurement strength, η is the measurement efficiency, and dW is the Wiener noise. The control parameters can then be updated in response to the measurement results.

Applying such a feedback-control formalism, Smith et al. realized an experimental set-up of an atom beam in a driven cavity and successfully stabilized to a conditional state by altering the strength of the drive field in response to the measured photon number on an avalanche photodiode [24]. Measured photon counts are used to form a $g^{(2)}$ correlation function which conditions an electro-optical modulator in front of a polarizer, altering the driving intensity of the cavity. Figure 7 demonstrates the effect of such feedback on the correlation function of the system over time. Whereas the system without feedback

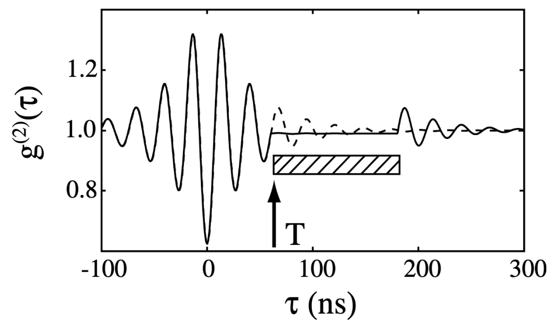

FIG. 7. Correlation function of the cavity QED system with feedback (continuous line) and without feedback (dashed line). Figure taken from Smith et al.

holds time-reversal symmetry, the system with feedback arrive at a new steady state dictated by the application of conditioning drive pulses.

Kohler et al. also employ a similar feedback mechanism, conditioning the collective spin of an atomic ensemble undergoing Larmor precession in an optical cavity [25]. In such a system, spin precession acts back on the probe field by way of amplitude modulation. Such fluctuations autonomously move the collective spin of the system to poles which can be altered by changing the detuning of the probe field. These effects are observed through a heterodyne detection of the cavity field, the results of which are shown in figure 8.

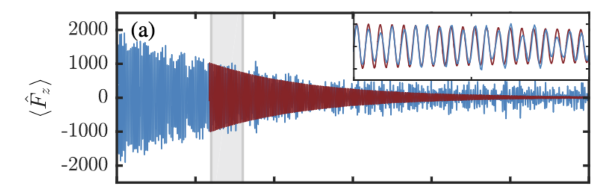

FIG. 8. Larmor precession of collective spins of the system moving towards steady state following displacement. Figure taken from Kohler et al.

Murch et al. apply this measurement-based feedback routine in the superconducting qubits, stabilizing the Rabi oscillations of the qubit in response to microwave photons in a readout cavity [26]. By dispersively coupling a transmon qubit to a readout cavity and performing continuous measurement of the homodyne signal, the microwave drive on the transmon qubit is tuned so that the qubit is stabilized to a desired Rabi oscillation frequency, as shown in figure 9. In a system without feedback, the Rabi oscillations gradually decay due to environmental decoherence and measurement-induced decoherence, but the presence of feedback enables the system to stabilize to a desired Rabi frequency.

Similar feedback techniques have been applied or proposed for a variety of use-cases, including quantum simulation of spin bath problems and Floquet time crystals

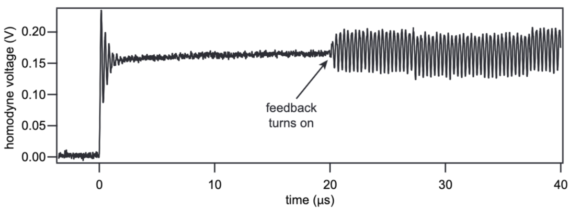

FIG. 9. Stabilization of Rabi oscillations of a transmon qubit in response to feedback. Figure taken from Murch et al. [27], probabilistic error correction [28], and metrology [29].

VI. CONCLUSIONS AND OUTLOOKS

In this review, I have discussed some basic concepts of cavity QED, measurement theory, and weak measurement. I have reviewed a series of works that have applied cavity weak measurement both as a means to understand the dynamics of quantum systems and to perform quantum feedback and control. The ability to perform continuous weak measurements on quantum systems has enabled a variety of applications, from stabilization of quantum systems to quantum error correction. With the maturation of quantum systems and the development of sophisticated fine-grain control techniques such as optical tweezers [30] and optical lattices [31], such feedback-control systems will likely become ubiquitous in the quantum devices of the future.

Furthermore, weak measurements can be applied to a myriad of other areas, such as the real-time observation of quantum dynamics and phase transitions [32], as well as investigations into the fundamental physics of effects such as decoherence and backaction [33–35]. There is a rich variety of physics that can be enabled by such techniques, and the exploration of cavity-assisted weak measurement represents a fascinating frontier in quantum science with profound implications for both fundamental physics and practical applications.

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