

Atomic bomb approximation

You

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1 Problem statement

- Atomic bomb frenzy: each nation wants one
- The U.S. has shown some pictures of bomb
- We want to figure out how strong the bomb actually is

What are the characteristics of the bomb we care about?

- Size: Volume, radius, etc.
- How quickly does it expand?
- How much energy is released?

Primarily interested in energy released : this gives us info on strength of bomb

How do we figure out the energy released from just four pictures and some timestamps? What information do we have?

LL: EXTRACTING INFORMATION FROM OBSERVATION

2 Energy

What is energy? How do we define it?

Two types, kinetic and potential:

- Kinetic: Arises from moving things: what work can they do? Let's think of some examples.
- Potential: Stored energy: what could this physical configuration do? (Gravitational potential, electric potential, etc.)

Mathematical Definitions

- $KE = \frac{1}{2}mv^2$
- $PE = mgh$

3 Dimensional Analysis

What are the dimensions of something

How is your height measured? How is your weight measured? What exactly are dimensions?

Measurement systems: there are lots of ways things can be measured. SI units, imperial units, atomic units, etc. **Emphasize that measurements need to be self consistent.**

Do some basic examples with areas and length and volumes and then distance and velocity and acceleration

What are the dimensions of energy? Let's just look at KE as an example: Break down into more basic units (let's just use SI units for all of these)

$$\text{Energy} = \frac{kg \cdot m^2}{s^2}$$

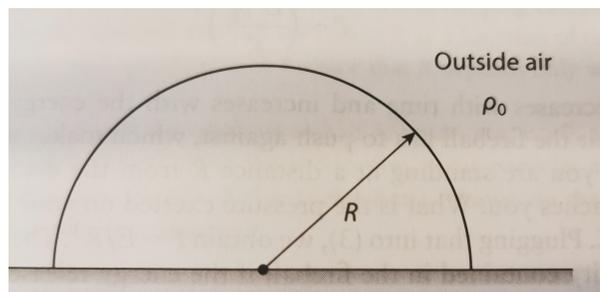
Let's also do some more practice. Are kinetic and potential energy consistent?

A preview: Let's check the units on density.

LL: CHECKING FOR CONSISTENCY

4 Breaking Down the Problem

Let's first simplify things: fireball is a sphere, fireball is purely surrounded by air. Schematically, we can think of it this way:



LL: MAKING SIMPLIFYING ASSUMPTIONS

It looks like we will need some sort of relationship between radius of fireball (R) and time (T)

We have:

1. Radius of fireball
2. Time measurement was made
3. Density of surrounding medium

Surrounding medium is air: **What are dimensions of density?**

$$D = M/V$$

$$\text{mass} = M$$

$$\text{volume} = L^3$$

$$[D] = \frac{M}{L^3}$$

So how should all these quantities relate?

We will think about the radius of the fireball in relation to the time, since that is the primary data we have in these images. Let's see how we can relate them with the tools we have:

$$1. [D] = \frac{M}{L^3}$$

$$2. [E] = \frac{ML^2}{T^2}$$

Let's not think too hard and just try to match units:

$$R = \left(\frac{Et^2}{D} \right)^{1/5} \quad (1)$$

This looks like this:

$$[R] = L = \left(\frac{ML^2/T^2 \times T^2}{M/L^3} \right)^{1/5} \quad (2)$$

Let's check our equation with limits:

What happens with R, does it grow with time as we expect?

As E gets bigger, is R getting bigger as we expect?

As D gets bigger, the fireball has to push harder to move the air, so R should be less. is this true?

5 Let's see if we can check how well we did

Let's try and plot this. Exponent is a bit messy so let's turn it into a log log plot.

Some quick log basics:

$$\log(a * b) = \log a + \log b \quad (3)$$

$$\log(a/b) = \log a - \log b \quad (4)$$

$$\log(a^2) = 2 \log a \quad (5)$$

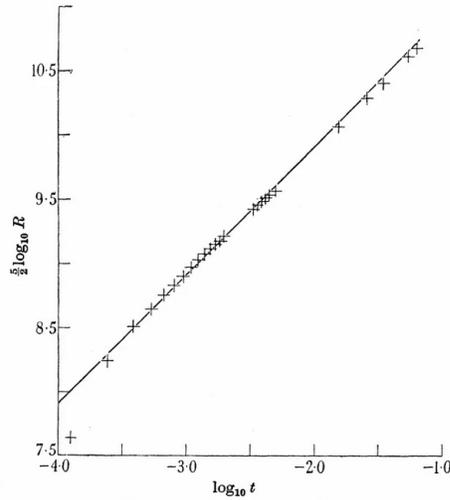
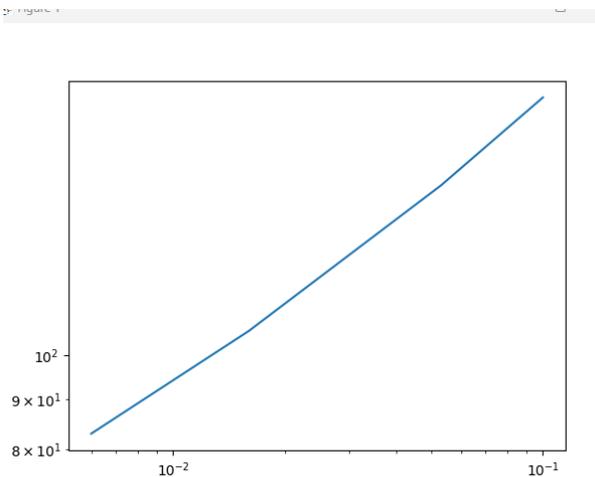


FIGURE 1. Logarithmic plot showing that $R^{\frac{2}{5}}$ is proportional to t .

$$\log R = \frac{2}{5} \log t + \frac{1}{5}(\log E - \log D) \quad (6)$$

Without even plotting it, we see slope of $2/5$ makes sense for time. Let's ignore the rest and just look at radius vs. time, since that is the primary information we have from the photos:

0.006 sec: R 83m
 0.016 sec: R 106m
 0.053 sec: R 150m
 0.100 sec: R 184m



We can also find energy to be:

$$E = \frac{DR^5}{t^2} \quad (7)$$

$D = 1.2 \text{ kg/m}^3$ for air. For first photo, $t = 0.015\text{s}$, $R = 106\text{m}$, $E = 7.14 * 10^{14}$ Joules. This is about 18 kilotons of TNT. The actual blast of the first bomb was equivalent to about 20 kilotons of TNT.

Comment about nuclear bomb, atomic physics, but really the central idea here is to just produce energy.